Section 6.5 Solutions

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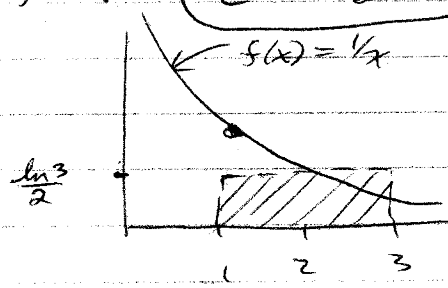
6. $f_{ave} = \frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} \sec^2\left(\frac{\theta}{2}\right)d\theta = \frac{2}{\pi}\left[2\tan\left(\frac{\theta}{2}\right)\right]\Big|_0^{\pi/2}$

$= \frac{4}{\pi}\left[\tan\left(\frac{\pi}{4}\right) - \tan(0)\right] = \frac{4}{\pi}\left[1-0\right] = \boxed{\frac{4}{\pi}}$

10. a) $f_{ave} = \frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx = \frac{1}{2}\left[\ln x\right]\Big|_1^3 = \boxed{\frac{1}{2}\ln 3}$

b) $f(c) = \frac{1}{c} = \frac{1}{2}\ln 3 = f_{ave}$, so $\boxed{c = \frac{2}{\ln 3}}$

c) 

18. $v(r) = \frac{P}{4\eta l}(R^2 - r^2)$, $\quad 0 \le r \le R$

$v_{ave} = \frac{1}{R-0}\int_0^R \frac{P}{4\eta l}(R^2-r^2)dr = \frac{1}{R}\cdot\frac{P}{4\eta l}\int_0^R (R^2-r^2)dr$

$= \frac{P}{R\cdot 4\eta l}\left[R^2 r - \frac{1}{3}r^3\right]\Big|_0^R = \frac{P}{4R\eta l}\cdot\frac{2}{3}R^3$

$v_{ave} = \boxed{\frac{PR^2}{6\eta l}}$

$\frac{d\,v(r)}{dr} = -\frac{2rP}{4\eta l} = -\frac{P}{2\eta l}r$

Since the derivative of $v(r)$ is negative throughout the interval $0 \le r \le R$, $v(r)$ must reach its maximum value when $r = 0$.

Thus, $v_{max}(r) = v(0) = \boxed{\frac{PR^2}{4\eta l}}$

Note that $v_{ave} = \frac{2}{3}v_{max}$